BLACK HOLES AND SINGULARITY THEOREMS SOMMERAKADEMIE 2013

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Internatsschule Schloss Neubeuern, August 4-17, 2013.

1. Course content

This is a rough plan for the Sommerakadamie. The material is divided in sections of 1 or 2 days. The participants listed for those days shall together prepare and present the material, with a rough time schedule of three hours per day. Do not hesitate to ask the course leaders for assistance. This holds for all of you, but in particular for those who get a large amount a material to present. Those presenting the first days must prepare the talks before arriving at the Sommerakademie. Please contact the course leaders as soon as you have a first draft of the content of your talks.

Some words about the language: the talks (except the talks by Mattias Dahl) will be in German. All organizers understand German, and probably all participants as well. However this plan will be in English.

1.1. **Background reading.** To be read by the students before the Sommerakademie. In case you do not have access to the literature, please send an email to the organizers.

Introduction. Some background on space, time, special relativity [Kri99, Chap. 1], [Wal84, Chap. 1]. Also some popular background on general relativity and cosmology.

Manifolds and Tensors. Background on differential manifolds, tensor fields. [Kri99, 2.1-2.3], [Wal84, Chap. 2], [Bära, Chap. 1]

Curvature. The Sommerakademie starts with curvature, but have a look at curvature and covariant derivative also before (for example [Bära, Chap. 2-3]), as we will have to proceed rapidly, and it is better if you have some time to digest these concepts.

1.2. **Curvature and Einstein's equations.** General relativity is formulated through an equation for the curvature of a Lorentizan metric on a smooth manifold. The simplest solution describing a black hole is the Schwarzschild metric.

Date: December 11, 2020.

Day 1-2. **Semi-riemannian manifolds and curvature.** Primarily, [Kri99, 2.7-2.9 and 4.3-4.6]. See also [Wal84, Chap. 3], [O'N83, Chap. 3], [Gal, Sec. 1], [Bära, Chap. 2-3].

Day 2-3. **Einstein's equations.** Primarily [Kri99, Chap. 5], see also [Wal84, Chap. 4], [O'N83, Chap. 12].

Day 4. **The Schwarzschild solution.** Primarily [Kri99, Chap. 7] and [Wal84, Chap. 6], see also [O'N83, Chap. 13], [MTW73, Chap. 31-32].

1.3. **Causal structure and singularity theorems.** The causal structure of a space-time describes the space and time relations between events, but without specific measurements of distances or time passed. The singularity theorems of Hawking and Penrose tell us that natural conditions on the matter imply that the space-time must have singularities.

Day 5-6. **Causal structure.** Primarily [Kri99, Chap. 8], see also [Wal84, Chap. 8], [O'N83, Chap. 14, to p431], [Gal, Sec. 2], [Bärb, Sec. 2.1-2.7].

Day 7-8. **Singularity theorems.** Primarily [Kri99, Chap. 9], see also [Wal84, Chap. 9], [O'N83, Chap. 14, from p431], [Gal, Sec. 3-4], [Bärb, Sec. 2.8-2.9].

1.4. **Black holes.** A black hole in a space-time is the region where light rays cannot escape to infinity. The cosmic censorship conjecture states that singularities must always be inside black holes.

Day 9-10. **Black holes.** *Ammann, Dahl.* Follow 12.1-12.2 of [Wal84], with some background from Chapter 11. See also [HE73, Chap. 9.1-9.2]. [MTW73, Chap. 33-34], [Chr], [Goub], [Goua]

- Classification of stationary black holes, "no hair"-theorems.
- The topology of black holes, Hawking's black hole topology theorem
- Cosmic censorship.

References

[Bära] Christian Bär, *Differentialgeometrie*, `http://geometrie.math.uni-potsdam.de/documents/baer/skripte/skript-DiffGeoErw.pdf`.

[Bärb] ———, *Lorentzgeometrie*, `http://geometrie.math.uni-potsdam.de/documents/baer/skripte/skript-LorGeo.pdf`.

[Chr] Piotr T. Chruściel, *Black holes as a mathematical problem*, Slides.

[Gal] Gregory J. Galloway, *Spacetime geometry*, `http://www.math.miami.edu/~galloway/beijing.pdf`.

[Goua] Éric Gourgoulhon, *Black hole physics entering a new observational era*, `http://luth.obspm.fr/~luthier/gourgoulhon/fr/present_rec/tours12.pdf`.

[Goub] ______, *Black holes: from event horizons to trapping horizons*, http://luth.obspm.fr/~luthier/gourgoulhon/fr/present_rec/cern10.pdf.

[HE73] S. W. Hawking and G. F. R. Ellis, *The large scale structure of space-time*, Cambridge University Press, London, 1973, Cambridge Monographs on Mathematical Physics, No. 1.

[Kri99] Marcus Kriele, *Spacetime, foundations of general relativity and differential geometry*, Lecture Notes in Physics. New Series m: Monographs, vol. 59, Springer-Verlag, Berlin, 1999.

[MTW73] Charles W. Misner, Kip S. Thorne, and John Archibald Wheeler, *Gravitation*, W. H. Freeman and Co., San Francisco, Calif., 1973.

[O'N83] Barrett O'Neill, *Semi-Riemannian geometry, with applications to relativity*, Pure and Applied Mathematics, vol. 103, Academic Press Inc. [Harcourt Brace Jovanovich Publishers], New York, 1983.

[Wal84] Robert M. Wald, *General relativity*, University of Chicago Press, Chicago, IL, 1984.

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