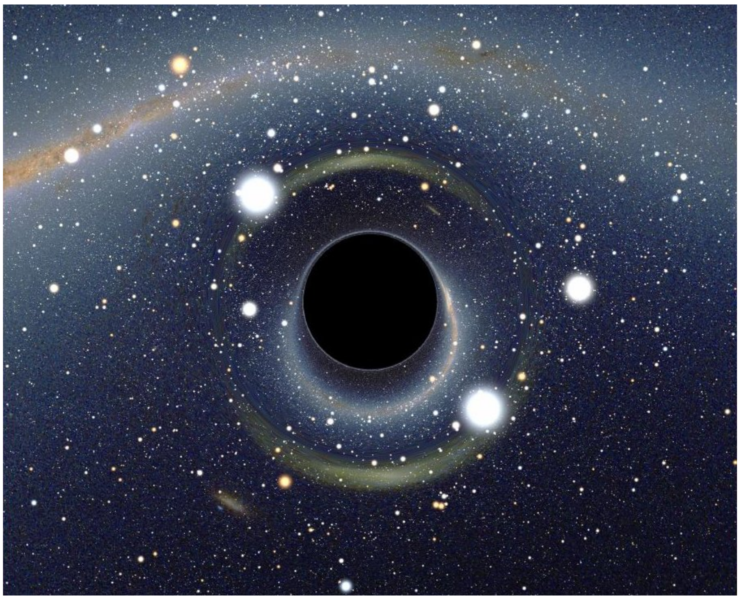
Differential Geometry II
Lorentzian Geometry
Lecture Notes



Prof. Dr. Bernd Ammann

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University of Regensburg

Presentation Version

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3.2.5

~~3.5~~ Cauchy hypersurfaces

Let A be achronal, i.e. no timelike f.-d. curve from A to A.

$$\operatorname{edge}(A) := \left\{ p \in \overline{A} \;\middle|\; \begin{array}{l} \text{for all open neighborhoods } U \text{ of } p \text{, there} \\ \text{is a f.-d. timelike curve in } U \text{ from } I_-^U(p) \\ \text{to } I_+^U(p) \text{ that does not hit } A. \end{array} \right\}$$

Definition 3.~~5~~2.69. *A subset $S \subset M$, $\dim M = m$ is a* **topological hypersurface**, *if for any $p \in S$ we can choose $U \Subset M$ with $p \in U$, $V \Subset \mathbb{R}^m$, and a homeomorphism $\varphi : U \to V$ such that*

$$\varphi(U \cap S) = V \cap \left(\{0\} \times \mathbb{R}^{m-1}\right).$$

Such a triple (U, V, φ), alternatively written as $U \xrightarrow{\varphi} V$, is called a **topological submanifold chart** *for S around p.*

Lemma 3.2.70a. *Let $S \subset M$, $\dim M = n+1$ be achronal and a topological hypersurface, then for any $p \in S$ we can choose a topological submanifold chart $U \xrightarrow{\varphi} V \subset \mathbb{R}^m \cong \mathbb{R}^{n,1}$ for S around p with the additional property that for any $(t_0, x) \in V$ the curve $t \mapsto \varphi^{-1}(t, x)$ is a smooth timelike curve, defined for $t \in (t_0 - \epsilon, t_0 + \epsilon)$ for some $\epsilon > 0$.*

A achronal $\iff$ there are no timelike f.d. curves from A to A