Where we are

M Lorentzian mfd, time-oriented

$P \subset M$ spacelike submfd. $q \in M$.

$q \in J_+(P) \iff \exists$ future-directed causal curve from P to q (or $q \in P$)

$q \in I_+(P) \iff \exists$ future-directed timelike curve from P to q.

$I_+(P) \subset J_+(P)$ Converse?

Vaguely $q \in J_+(P) \setminus P \overset{?}{\Longrightarrow} q \in I_+(P)$

Previously $P = \{p\}$, c a future-directed causal curve from p to q

$\langle \dot{c}(t), \dot{c}(t) \rangle \geq 0$. Can we get $\langle \dot{\tilde{c}}(t), \dot{\tilde{c}}(t) \rangle > 0$?

Then there is a future-directed timelike curve from p to q (close to c) unless c is a lightlike geodesic.

<u>Now</u>: $\dim P > 0$. Allow to vary the start point (locally), $p \in P$.
Is there a future-directed timelike curve from P to q close to c?

1) Prop 2.38 $\dot{c}(0) \perp T_{c(0)}P$

2) Lemma 2.43 $\dot{c}(0) \not\perp T_{c(0)}P$.

<u>Riemannian analogue</u>