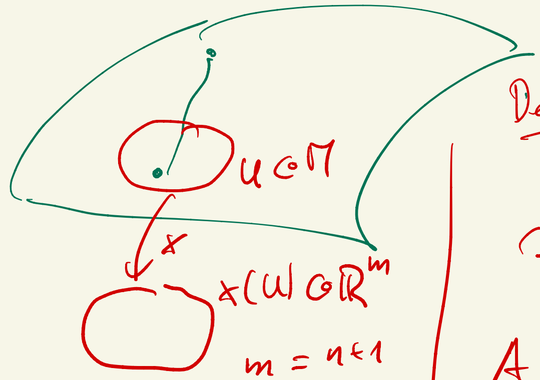
Causality Kausalität

C. Bär Script on Lorentzian geometry



$x = (x^0, \dots, x^n)$

Def 2.1 Let (M,g) be a Lorentzian manifold.

$$P(TM) = \bigcup_{p \in M} P(T_pM)$$

A time orientation is a map $\xi : M \to P(TM)$ such that for all $p \in M$

(1) $\xi(p)$ is one of the two connected components $I_\pm(T_pM)$ of $\{x \in T_pM \mid g(x,x) < 0\}$

(2) there is a chart (x,U) around p such that $\frac{\partial}{\partial x^0}\Big|_q \in \xi(p)$ $\forall q \in U$.

We say (M,g) is time-orientable, if such a ξ exists.

(M,g,ξ) is called a time-oriented Lorentzian manifold

Prop TFAE

(i) M is time-orientable

(ii) M admits a continuous timelike vector field

(iii) M admits a smooth time like vector field

Proof (iii) $\Rightarrow$ (ii) ✓

(ii) Let X be a cont. timelike vector field on M. For $p \in M$ choose a chart such that $\frac{\partial}{\partial x^0}\Big|_p$ is time like.

By possibly replacing x^0 by $-x^0$ we can achieve that $\frac{\partial}{\partial x^0}\big|_p$ and $X_{|p}$ are in the same connected component of timelike vectors,

$\mathfrak{f}(p) := $ connected comp. of $X_{|p}$

$\frac{\partial}{\partial x^0}\big|_q \in \mathfrak{f}(q)$ by continuity of g and X.

$\Leftrightarrow$ $g\left(\frac{\partial}{\partial x^0}\big|_q, X_q\right) < 0$

$\Rightarrow \mathfrak{f}$ is a time-orient.