

Seminar: Analytic K -homology

Summer term 2019

Prof. Bernd Ammann

Tuesday 14-16, (excluding holidays etc, there are 12 weeks available)

1 Aim of the seminar

In the seminar we develop an analytic approach for K -homology. This establishes links between Riemannian geometry, algebraic topology and functional analysis. The origins of the area lie on the one hand side in the theory of linear operators on Hilbert spaces, and on the other side on the Atiyah-Singer index theorem which already provides a method to calculate the Fredholm index of an elliptic operator in terms of characteristic classes from algebraic topology. One goal of the seminar is an analytic definition of Kasparov's K -homology and the associated index pairing. K -homology can also be defined abstractly as the homology theory dual to the generalized cohomology theory given by Atiyah-Hirzebruch K -theory. Thus our seminar provides a concrete and geometric picture of this homology theory, establishing a helpful link to operator theory.

An important result in the seminar will be the Brown-Douglas-Fillmore theorem which states that two essentially normal operators T_1 and T_2 with the same essential spectrum X are essentially unitarily equivalent if and only if $\text{Index}(T_1 - \lambda \text{id}) = \text{Index}(T_2 - \lambda \text{id})$ for every $\lambda \in \mathbb{C} \setminus X$.

The seminar then discusses the Kasparov product, KK -theory, further links to the classical Atiyah-Singer index theorem, index theory for hypersurfaces, higher index theory and obstructions against metrics of positive scalar curvature.

2 Talks

We will mainly follow a textbook written by Nigel Higson and John Roe [4]. Almost all chapters have about the same size and probably contain a bit more information than what can be presented in one session. Additionally each chapter contains some very helpful exercise. We thus reserve two sessions per chapter, but the speaker should leave some time to discuss the exercises (and choose some exercises as homework for the audience).

The seminar already started in the winter term, treating the following topics, see [4, Chap. 1–3]

- C^* -algebras and operator theory
- Index theory and extensions
- Completely positive maps and Voiculescu's theorem

- Kasparov's Technical Theorem

Talk no. 1: *K*-Theory. 30.4. + 14.5. + 21.5. JULIAN SEIPEL.

The talk should follow [4, Sec. 4]. Additional information can be obtained in [5] and [3].

(No talk on 7.5.)

Talk no. 2: Coarse Geometry and *K*-Homology, Part I. 27.5. + 5.6.

ALEXANDER ENGEL.

Coarse structures are a tool to discuss the global structure of non-compact spaces, in a way which is robust under local (e.g. compact) changes of the space. In this talk we discuss coarse structures, their relation to metric structures and the associated *K*-theory [4, Sec. 6, first part].

Talk no. 3: Duality Theory. 18.6. + 25.6. GUADALUPE CASTILLO SOLANO.

Discuss the dual *C**-algebra associated to a representation on a Hilbert space. This allows us to see extension groups as *K*-groups (Prop 5.1.6). We now can define (relative) *K*-homology and discuss its basic properties [4, Sec. 5].

Talk no. 4: Coarse Geometry and *K*-Homology, Part II. 2.7. + 9.7.

ALEXANDER ENGEL. The main goal of this talk is to establish a relationship between the *K*-theories defined in the previous two talks. In particular, we want to show

$$K_p(C^*(X)) \cong \tilde{K}^{1-p}(C(\partial X))$$

for $\partial X := \overline{X} \setminus X$ where \overline{X} is a metrizable compactification of the locally compact space X . We follow [4, Sec. 6, second part].

Talk no. 5: The Brown-Douglas-Fillmore theorem. 16.7. + 22.7. JONATHAN GLÖCKLE., 23.7. ALEXANDER ENGEL.

In this talk we prove the Brown-Douglas-Fillmore theorem (stated above) which is one of the most important results of the seminar. [4, Sec. 7]. It is a consequence of an index map

$$\text{Index} : K^p(C(X)) \rightarrow \text{Hom}(K_p(C(X)), \mathbb{Z})$$

which is an isomorphism for all p .

Seminar-Homepage

<http://www.mathematik.uni-regensburg.de/ammann/k-homology2>

Literatur

- [1] Bruce Blackadar; *K*-Theory for Operator Algebras; MSRI Research Publications

- [2] Joachim Cuntz, Ralf Meyer, Jonathan Rosenberg; Topological and Bivariant K -Theory; Oberwolfach Seminars **36**; Birkhäuser
- [3] Thomas Friedrich; Vorlesungen über K -Theorie; B. G. Teubner-Verlagsgesellschaft, Leipzig; 1978
- [4] Nigel Higson, John Roe; Analytic K -Homology; Oxford Mathematical Monographs; Oxford University Press
- [5] M. Rørdam, F. Larsen, N. J. Laustsen; An Introduction to K -Theory for C^* -Algebras; London Mathematical Society Student Texts **49**