Seminar: Kähler manifolds

Winter term 2017/18

Prof. Bernd Ammann

Monday 16-18
Special dates:

- 23.10. Gauß lecture

- 11.12. GK meeting in Windberg

1 Complex manifolds and complex geometry

This part is the center of the seminar. Here the participants should learn the basic definitions, constructions and results for complex manifolds

Talk no. 1: Complex and holomorphic structures. *16.10.*
[2, Chap. 7 and 8].

Talk no. 2: Vector bundles with complex, holomorphic and hermitian structures. *30.10. + 6.11.*
[2, Chap. 9 and 10]

Talk no. 3: Hermitian and Kähler metrics. *13.11.*
[2, Chap. 11 and Chap. 12]

Talk no. 4: Curvature of Kähler metrics and examples. *20.11.*
[2, Chap. 13]

Talk no. 5: Laplace operators, Hodge and Dolbeault theories. *27.11. + 4.12.*
[2, Chap. 14 and 15]
Discussion about Part III *18.12.*

2 Advanced topics

Talk no. 6: Ricci curvature as curvature of the canonical line bundle. *8.1.*
The talk essentially coincides with [2, Chap. 17]. We assume that the readers already know Chern clases (as in [2, Chap. 16]), but the properties used in the talk should be recalled when needed. The most important statement in the talk is Theorem 17.5. If the speaker has enough time during preparation, it would be helpful to discuss which of the equivalences remain true for non-simply-connected manifolds.

Talk no. 7: Calabi-Yau manifolds and Aubin-Yau theorem. *15.12.*
The talk should sketch Calabi's idea to construct a Ricci-flat Kähler metric on each Kähler class on a complex manifold with vanishing first chern class. The speaker should follow [2, Chap. 18]. The hard part in this construction are a priori estimates due to Yau, but this should **not** be discussed (similar to [2]). The speaker then should explain how to modify this construction to obtain the Aubin-Yau theorem, follow [2, Sec. 19.1].

Talk no. 8: Vanishing results. *22.1.*
Weitzenböck formulas should be established. Then it should be proved how various assumption on Ricci curvature imply that holomorphic vector fields or holomorphic volume forms do not exist. This yields obstructions to metrics with such curvatures. [2, Chap. 20]. This talk could be omitted, or dealt later on, and then one could just cite its results in the next talk.

Talk no. 9: The Hirzebruch-Riemann-Roch formula. *29.1.*
In this talk we discuss the Hirzebruch-Riemann-Roch formula as a special case of the Atiyah-Singer index theorem. The talk should be given by someone who is already familiar with the Atiyah-Singer index theorem, to combine this with Moroianu's book. See [2, Chap. 21]

Seminar-Homepage

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http://www.mathematik.uni-regensburg.de/ammann/kaehler
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Literatur

[1] BALLMANN, W. Lectures on Kähler manifolds. ESI Lectures in Mathematics and Physics. European Mathematical Society (EMS), Zürich, 2006.

[2] MOROIANU, A. Lectures on Kähler geometry, vol. 69 of London Mathematical Society Student Texts. Cambridge University Press, Cambridge, 2007.