Enlargeability

Universität Regensburg, Wintersemester 2012/13

1. 8.1.2012 *Enlargeability and obstructions to positive scalar curvature*, [3, IV, §5 until Theorem 5.12]. Explain the definition of enlargeability, the basic properties, and show that it is an obstruction to the existence of a positive scalar curvature metric. The Atiyah-Singer index theorem should be briefly recalled for this purpose, clearly without proof. Then explain how to generalize enlargeability to $\hat{A}$-enlargeability and prove the related obstruction to positive scalar curvature.

2. 15.1.2012 *Variations of enlargeability*, [3, IV, §6]. Explain the relative index theorem for complete manifolds with positive scalar curvature at infinity (Theorem IV.6.5 of [3]). Introduce weak enlargeability and explain how to get important examples (IV.6.10 to IV.6.12). Show that weak enlargeability is already an obstruction to positive scalar curvature Theorem IV.6.13. Discuss applications (IV.6.14 to IV.6.17).

3. 22.1.2012 *Classification of orientable* 3*-manifolds admitting positive scalar curvature* Proof [3, Theorem IV.6.18], the original can be found in [2, Sec. 7]. Discuss the classification of 3-manifolds admitting positive scalar curvature. Note that [3] was written before Perelman's work on Ricci flow and geometrization, Lawson and Michelsohn call it "standard conjectures". It follows from this paragraph in [3] that an orientable 3-manifold with positive scalar curvature cannot have a $K(\pi, 1)$-factor in its prime decomposition. Now recent results on geometrization yield that the manifolds Σ_i in [3, IV. §6] are in fact quotients of S^3 by discrete subroups of $SO(4)$ acting freely on S^3, see e.g. the left column, page 1230 of [4] or other standard references on geometrization via Ricci flow. In combination with [3, Proposition IV.4.3] give a classification of all orientable 3-manifolds admitting positive scalar curvature metrics.
   Note: The subgroups of $SO(4)$ acting freely on S^3 can be explicitely classified, see [1, Ch. 4]. The existence of this classification should be mentioned, but as the classification has many subcases, it should not be presented in the talk. The fact that a classification exists relies on the classification of the discrete subgroups of $SU(2)$, see [5, Page 87–88, Theorem 2.6.7], together with $SO(4) = SU(2) \times SU(2)/\pm 1$.

4. 29.1.2012 *Enlargeability as a Mishchenko index*We follow the overview article arXiv:1011.3987 by Bernhard Hanke. Related literature by Hanke, Schick, Kotschick, Roe, Brunnbauer: arXiv:0707.1999, arXiv:math/0604540, arXiv:math/0403257, arxiv.org/abs/0902.0869

Literatur

[1] J.H. Conway, D.A. Smith, *On quaternions and octonions: their geometry, arithmetic, and symmetry*, A K Peters, 2003.

[2] M. Gromov, H.B. Lawson, *Positive scalar curvature and the Dirac operator on complete Riemannian manifolds*, Inst. Hautes Études Sci. Publ. Math. No. **58** (1983), 83–196.

[3] H. B. Lawson and M.-L. Michelsohn, *Spin geometry*, Princeton University Press, Princeton, 1989.

[4] J. Milnor, *Towards the Poincaré conjecture and the classification of 3-manifolds*, Notices Amer. Math. Soc. **50** (2003), no. 10, 1226—1233.

[5] J. A. Wolf, *Spaces of constant curvature*, Mc Graw Hill, 1967