Geometric Analysis
Regensburg, March 7–11, 2023

The electronic version of this program is interactive: you may click to names and titles and get to the abstract or to the schedule. References to the arxiv are links.

Talks on Tuesday, March 7th

8:30–10:00 Registration

09:00–10:00 **Ilaria Mondello**

Gromov-Hausdorff limits of manifolds with a Kato bound on the Ricci curvature

10:00 Coffee break and Registration

10:30–11:00 **Jørgen Lye**

Singular Yamabe Flow

11:00 Coffee break

11:30–12:30 **Yoshihiko Matsumoto**

Renormalized energy of maps and conformal geodesics

Lunch

14:30–15:30 **Klaus Kröncke**

A new mass-type invariant for asymptotically hyperbolic manifolds

15:30 Coffee break

16:00–17:00 **Shota Hamanaka**

Some limit theorems for the total scalar curvature

Talks on Wednesday, March 8th

09:00–10:00 **Anna Siffert**

p-harmonic self-maps of spheres

10:00 Coffee break

10:30–11:00 **Lothar Schiemanowski**

Obstructions to the desingularization of nearly G_2 and nearly Kähler conifolds

11:00 Coffee break

11:30–12:30 **Sven Hirsch**

On a generalization of Geroch's conjecture

Lunch

14:30–15:30 **Demetre Kazaras**

Applications of (spacetime)harmonic functions

15:30 Coffee break

16:00–17:00 **Shinichiroh Matsuo**

Lattice gauge theory and the discretization of Dirac operators

Talks on Thursday, March 9th

09:00–10:00 **Julius Baldauf**

Harmonic spinors in the Ricci flow

10:00 Coffee break

10:20–11:20 **Olivier Biquard**

Dirac operators for Poincaré-Einstein metrics and their BGG boundaries

11:20 Coffee break

11:40–12:40 **Tristan Ozuch**

4-dimensional specific aspects of Ricci flows

Lunch

15:00 Guided Tour in Regensburg

19:00 Conference Dinner in the Restaurant "Leerer Beutel"

Talks on Friday, March 10th

09:00–10:00 **Hokuto Konno**

Homological instability for moduli spaces of 4-manifolds

10:00 Coffee break

10:30–11:00 **David Tewodrose**

Critical metrics of eigenvalue functionals via subdifferential

11:00 Coffee break

11:30–12:30 **Asma Hassannezhad**

Pleijel nodal domain theorem for the Robin eigenvalue problem

Lunch

14:30–15:00 **Jonathan Glöckle**

Initial data rigidity via Dirac-Witten operators

15:00 Coffee break

15:20–15:40 **Roger Tagne-Wafo**

Global Dynamics for inhomogeneous Maxwell-Boltzmann system in Robertson-Walker spacetime with Israel particles

15:40 Coffee break

16:15–17:15 **Tsuyoshi Kato**

Covering monopole map and aspherical 10/8-inequality conjecture

Talks on Saturday, March 11th

09:00–10:00 **Giovanni Catino**

Two rigidity results for stable minimal hypersurfaces

10:00 Coffee break

10:30–11:00 **Joseph Hoisington**

Calibrations and energy-minimizing mappings of rank-1 symmetric spaces

11:00 Coffee break

11:30–12:30 **Nadine Große**

On the L^p-spectrum of the Dirac operator

Lunch

End of the Conference

Abstracts

Julius Baldauf (MIT, USA)

Harmonic spinors in the Ricci flow

This talk will present the theory of spin geometry on weighted manifolds and its applications to the Ricci flow. It will be shown that Perelman's Ricci flow entropy can be expressed in terms of the energy of harmonic spinors, and in four dimensions, in terms of the energy of Seiberg–Witten monopoles. Consequently, Ricci flow is the gradient flow of these energies.

Olivier Biquard (Sorbonne University, France)

Dirac operators for Poincaré-Einstein metrics and their BGG boundaries

We show that the Atiyah-Schmid construction of discrete series via Dirac operators is related to the well-known BGG operators in conformal geometry. We deduce a new geometric construction of the BGG operators. Joint work with Robin Graham.

Giovanni Catino (Politecnico di Milano, Italy)

Two rigidity results for stable minimal hypersurfaces

In this talk I will describe two recent results concerning the rigidity of complete, immersed, orientable, stable minimal hypersurfaces: they are hyperplane in $\mathbb{R}^4$ while they do not exist in positively curved closed Riemannian (n+1)–manifold when $n \leq 5$. The first result was recently proved also by Chodosh and Li, and the second is a consequence of a more general result concerning minimal surfaces with finite index. Both theorems rely on a conformal method, inspired by a classical paper of Fischer–Colbrie. This is a joint work with P. Mastrolia (Università degli Studi di Milano) and A. Roncoroni (Politecnico di Milano).

Jonathan Glöckle (University of Regensburg, Germany)

Initial data rigidity via Dirac-Witten operators

Initial data sets (g, k) on a manifold M consist of a Riemannian metric g and a symmetric 2-tensor k. They typically arise in general relativity, when looking at a spacelike hypersurface M of a time-oriented Lorentzian manifold. In this case, g is the induced metric and k the induced second fundamental form. In this talk, we consider the situation where M is a manifold with boundary and (g, k) satisfies the dominant energy condition as well as a certain boundary condition. Using Dirac-Witten operators we prove a rigidity

theorem à la Eichmair–Galloway–Mendes (arXiv: 2010.06739) stating that M must be diffeomorphic to a cylinder $N \times [0,1]$ and is foliated by MOTS carrying non-trivial parallel spinors for the induced metrics. A special case of this is a rigidity statement for Riemannian metrics of non-negative scalar curvature and with mean convex boundary.

Nadine Große (University of Freiburg, Germany)

On the L^p-spectrum of the Dirac operator

We study the p-dependence of the spectrum of the Dirac operator. In particular, we give sufficient conditions for the L^p-spectrum to be independent on p on noncompact manifolds. As an application we use this result to compute the L^2-spectrum of classes of manifolds by calculating the L^1-spectrum. This is joint work with Nelia Charalambous.

Shota Hamanaka (Mitsubishi Electric Corporation, Advanced Technology R& D Center, Japan)

Some limit theorems for the total scalar curvature

This talk consists of two parts. In the first part, we show that the lower bound of the total scalar curvatures on a closed manifold is preserved under the C^0 or C^1 convergence of the Riemannian metrics under some assumptions. In the second part, we show that in a fixed conformal class on a closed manifold, the upper bound condition of the total scalar curvature is C^0-closed if its Yamabe constant is nonpositive. Moreover, we show that if a conformal class on a closed manifold has positive Yamabe constant, then, in such a conformal class, the condition that the scalar curvatures are bounded from below, as well as the total scalar curvatures are bounded from above is C^0-closed.This talk is based on my preprints: arXiv: 2208.01865 and arXiv: 2301.05444.

Asma Hassannezhad (University of Bristol, United Kingdom)

Pleijel nodal domain theorem for the Robin eigenvalue problem

A nodal domain refers to a connected region where the eigenfunction is nonzero. The simplest topological invariant of nodal domains is the nodal count. In this talk, we review the celebrated Courant and Pleijel nodal domain theorems and some of the recent developments in this direction. The main focus of the talk will be on an improved version of the Pleijel theorem for the Robin problem without restriction on the sign of the Robin parameter. This is joint work with David Sher.

Sven Hirsch (Duke University, USA)

On a generalization of Geroch's conjecture

The theorem of Bonnet-Myers implies that manifolds with topology $M^{n-1} \times S^1$ do not admit a metric of positive Ricci curvature, while the resolution of Geroch's conjecture shows that the torus T^n does not admit a metric of positive scalar curvature. In this talk I will introduce a new notion of curvature which interpolates between Ricci and scalar curvature (so-called m-intermediate curvature) and use stable weighted slicings to show that for $n \leq 7$ the manifolds $M^{n-m} \times T^m$ do not admit a metric of positive m-intermediate curvature. This is joint work with Simon Brendle and Florian Johne.

Joseph Hoisington (MPI Bonn, Germany)

Calibrations and energy-minimizing mappings of rank-1 symmetric spaces

We will prove lower bounds for energy functionals of mappings from real, complex and quaternionic projective spaces to Riemannian manifolds. For real and complex projective spaces these results are sharp, and we will characterize the family of mappings which minimize energy in these results. We will also discuss some connections between these results and several questions in systolic geometry.

Tsuyoshi Kato (Kyoto University, Japan)

Covering monopole map and aspherical 10/8-inequality conjecture

The aspherical 10/8-inequality conjecture is obtained by combining a covering 10/8-inequality conjecture and Singer conjecture. In this talk, I will present a survey of recent developments on related topics.

Demetre Kazaras (Duke University, USA)

Applications of (spacetime)harmonic functions

In a previous work, a new expression was obtained for the ADM mass of asymptotically Euclidean 3-dimensional initial data sets using so-called 'spacetimeharmonic functions.' Paralleling developments in minimal surface and Dirac operator approaches, these functions have proven to be useful tools in studying several pure-geometry questions. We will describe a selection of such applications, including a version of the Bonnet-Meyer diameter estimate for incomplete manifolds, black hole existence results, and progress on the Gromov and Sormani almost-rigidity conjectures.

Hokuto Konno (University of Tokyo, Japan)

Homological instability for moduli spaces of 4-manifolds

We prove that homological stability with respect to connected sums of S^2S^2 fails for moduli spaces BDiff(X) of simply-connected closed 4-manifolds X. This makes a striking contrast with other dimensions: in all even dimensions except for 4, analogous homological stability for moduli spaces has been established by work of Harer and of Galatius and Randal-Williams. The proof of the above result is based on a characteristic class constructed using the Seiberg-Witten equations. This is joint work with Jianfeng Lin.

Klaus Kröncke (KTH Stockholm, Sweden)

A new mass-type invariant for asymptotically hyperbolic manifolds

On asymptotically hyperbolic manifolds, we consider a particular linear combination of the renormalized volume and the boundary integral for the usual ADM mass. This quantity is well-defined and diffeomorphism invariant under weaker falloff conditions for the metric at infinity than one needs to define the renormalized volume and the hyperbolic ADM mass separately. We use this quantity to define a variant of the expander entropy on asymptotically hyperbolic manifolds which is monotonically increasing under the Ricci flow. Finally we use the expander entropy to prove a local positive mass theorem for Poincaré-Einstein metrics. This is joint work with Mattias Dahl and Stephen McCormick.

Jørgen Lye (University of Hannover, Germany)

Singular Yamabe Flow

I will talk about the Yamabe flow for positive Yamabe constant on a class of compact spaces including smoothly stratified spaces with iterated cone-edge metrics. The Yamabe flow exists for all time in this setting, but it might fail to converge. I will discuss this lack of convergence and present a result guaranteeing convergence. This is joint work with Gilles Carron and Boris Vertman, arXiv: 2106.01799.

Yoshihiko Matsumoto (Osaka University, Japan)

Renormalized energy of maps and conformal geodesics

Conformal geodesics, or *conformal circles*, are distinguished curves in a conformal manifold M satisfying a third-order ODE, which are defined in terms of the associated normal Cartan connection. Fine and Herfray recently found a characterization of conformal geodesics using proper minimal surfaces in

asymptotically hyperbolic Einstein spaces (Poincaré-Einstein spaces) (X, g_+) with boundary at infinity M. In this talk, I will reinterpret the Fine–Herfray characterization using proper harmonic maps targeted at (X, g_+). More specifically, Fine–Herfray discusses critical surfaces (constructed formally as power series) for the renormalized area functional $\mathcal{A}_{\mathrm{ren}}$, and I need to use critical maps (also constructed formally) for the renormalized energy $\mathcal{E}_{\mathrm{ren}}$ that I introduce. The coincidence of the two Neumann conditions deduced from criticalities with respect to $\mathcal{A}_{\mathrm{ren}}$ and $\mathcal{E}_{\mathrm{ren}}$ is nontrivial.

Shinichiroh Matsuo (Nagoya University, Japan)

Lattice gauge theory and the discretization of Dirac operators

Our ultimate goal is the discretization of Seiberg-Witten theory. As a first step towards this goal, we try to discretize the analytic index of Dirac operators. The analytic index of Fredholm operators is, however, a truly infinite dimensional phenomenon, and that of finite dimensional self-adjoint operators is always zero. Thus, a naive discretization of Dirac operators does not work. In this talk, I will show that the "Wilson-Dirac operator" considered in lattice gauge theory gives a correct discretization, at least from the viewpoint of the analytic index. This talk is based on a joint work with three physicists and two mathematicians: Hidenori Fukaya, Mikio Furuta, Tetsuya Oonogi, Satoshi Yamaguchi, and Mayuko Yamashita.

Ilaria Mondello (Université de Paris Est Créteil, France)

Gromov-Hausdorff limits of manifolds with a Kato bound on the Ricci curvature

In this talk I will present some recent results obtained in collaboration with G. Carron and D. Tewodrose about the structure of Gromov-Hausdorff limits of manifolds with Ricci curvature satisfying a Kato integral bound. This condition is implied for instance by a lower Ricci curvature bound, or an integral Ricci bound in the spirit of the work of Petersen-Wei. We focus in particular on the introduction of new almost monotone quantities based on the heat kernel, and their role in proving a regularity theory that recovers previous results by Cheeger and Colding.

Tristan Ozuch (MIT, USA)

4-dimensional specific aspects of Ricci flows

Ricci flow has been extensively studied, and most results are applicable either only in 3-dimensional or n-dimensional spaces. However, given the potential topological applications, a theory specific to the 4-dimensional situation is desirable. In this discussion, I will present tools and techniques that are

unique to the 4-dimensional case. Together with A. Deruelle, we introduce a notion of stability for orbifold singularities. This notion helps to explain the formation of orbifold singularities along Ricci flow. Moreover, in collaboration with K. Naff, we utilize self-duality in dimension 4 to simplify the evolution equations of curvature. This approach has enabled us to uncover intriguing connections between Ricci flow and Yang-Mills flow.

Lothar Schiemanowski (University of Kiel, Germany)

Obstructions to the desingularization of nearly G_2 and nearly Kähler conifolds

I will discuss the problem of desingularizing nearly Kähler and nearly G_2 manifolds with conical singularities using asymptotically conical Calabi–Yau 3-folds or G_2 manifolds. The first obstruction to such a desingularization turns out to be the exactness of certain forms on the AC manifolds. A rigidity result shows that below the rate threshold -3 for the AC manifold in question, this can only happen for Euclidean spaces.

Anna Siffert (University of Münster, Germany)

p-harmonic self-maps of spheres

We study rotationally p-harmonic self-maps between spheres. We prove that for $p \in \mathbb{N}$ given, there exist infinitely many p-harmonic self-maps of $\mathbb{S}^m$ for each $m \in \mathbb{N}$ with $p < m < 2 + p + 2\sqrt{p}$. Further, we show that for $p > m$, the identity map on $\mathbb{S}^m$ is stable when interpreted as a p-harmonic self-map of $\mathbb{S}^m$. This is joint work with Volker Branding.

Roger Tagne-Wafo (University of Douala, Cameroon)

Global Dynamics for inhomogeneous Maxwell-Boltzmann system in Robertson-Walker spacetime with Israel particles

In this talk we will discuss the global(in time) solutions for the Maxwell-Boltzmann system in the case where the background metric is that of Friedman-Lemaître-Robertson-Walker in the spatially inho-mogeneous case. The unknown functions depend not only on time but also on the space variables $(xi), i = 1; 2; 3$. We assume that the collision kernel is generated by Israel particles. By combining the energy estimates method with that of characteristics we derive, under hypotheses that the data are small in some appropriate norm, a unique global (in time) solution in suitable weighted spaces. Joint work with B. Benaja and E. Takou

David Tewodrose (University of Nantes, France)

Critical metrics of eigenvalue functionals via subdifferential

I will present a joint work with Romain Petrides (Université Paris Cité) where we propose a general approach to study critical metrics of functionals F from U to $R, F(g) = F(S_g)$, where U is an open set of Riemannian metrics on a given smooth manifold, S_g is a set of eigenvalues depending on g and F is a locally Lipschitz function. At the core of our approach is Clarke's notion of subdifferential. This covers well-known cases, like Laplace or Steklov eigenvalues, and provides new possibilities for other situations.